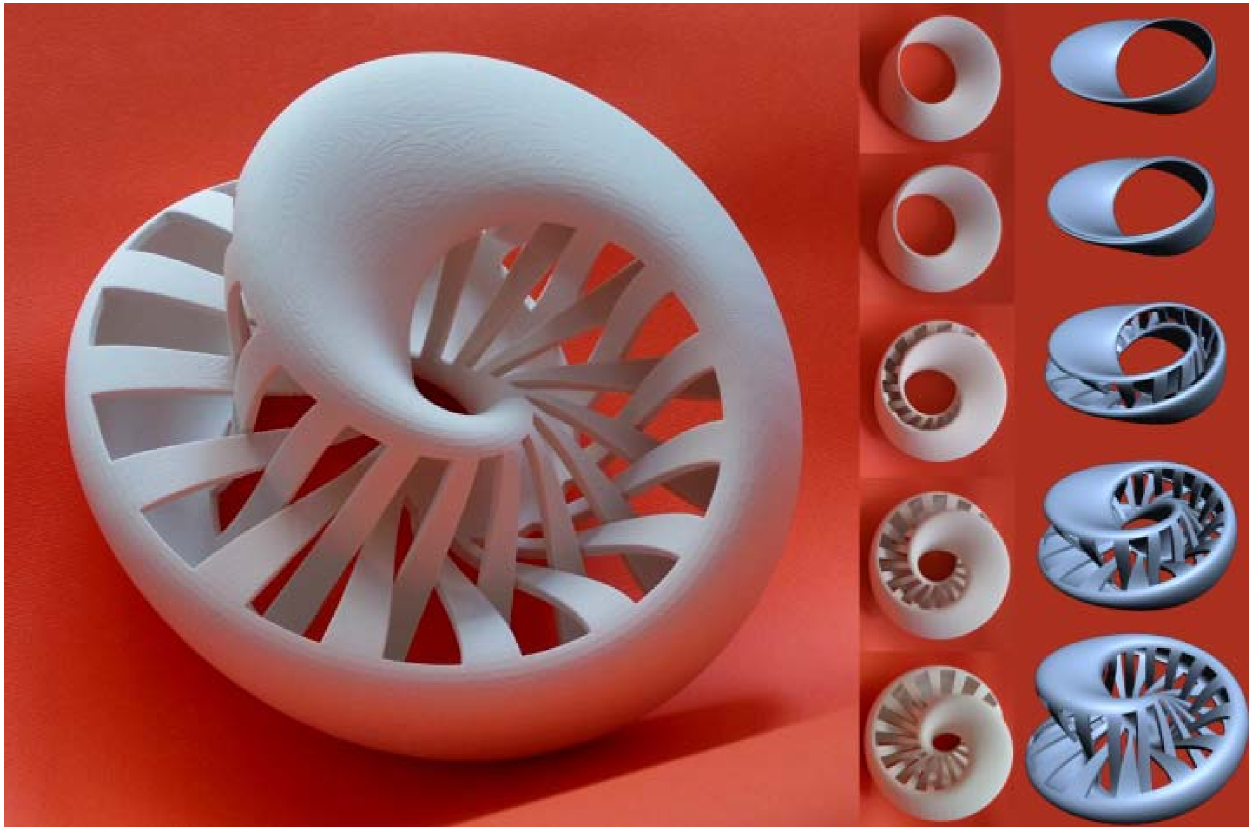
Models of surfaces by 3D printing



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Gregorio Franzoni – Università di Cagliari – Dip. di Matematica e Informatica

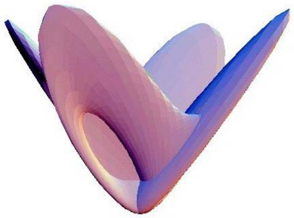
Models of mathematical objects

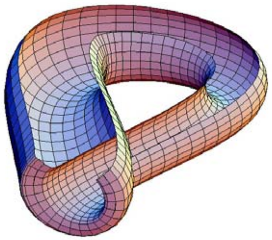
Why producing visual and physical models?

Fields of **material modeling** and of **computer visualization** of mathematical objects are strictly linked. Both belong to the more general context of their **representation**. Material models add the tactile experience to the visual one.

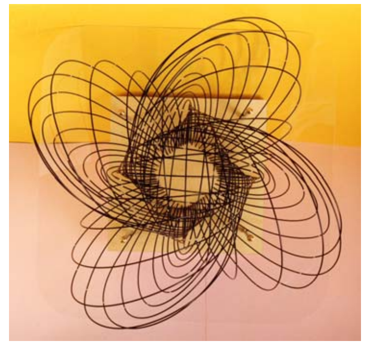
Aims of visualization and modeling:

- better understanding of abstract mathematical phenomena;
- help intuition in (more or less elementary) education;
- get new shapes from abstract research;
- popularization of Mathematics;
- applications in design field;
- establishing/discovering links between Science and Art;



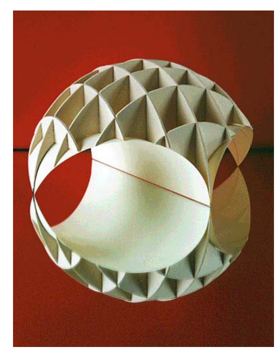


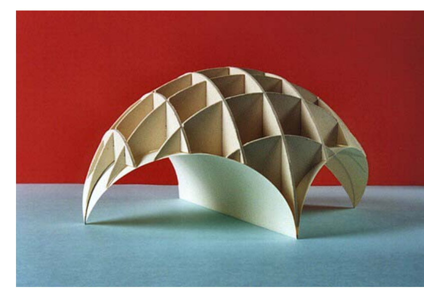
Material models of surfaces



Is it possible to realize material models of surfaces by using several techniques and materials.

In this presentation we mainly deal with *Rapid Prototyping* and *3D printing* techniques.





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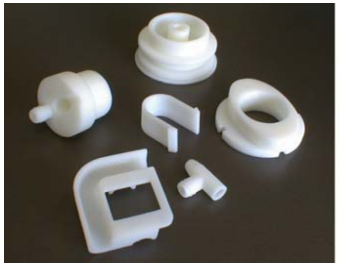
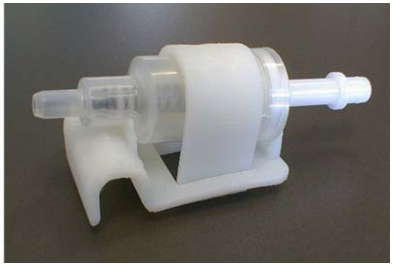
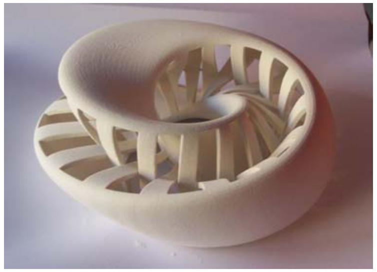
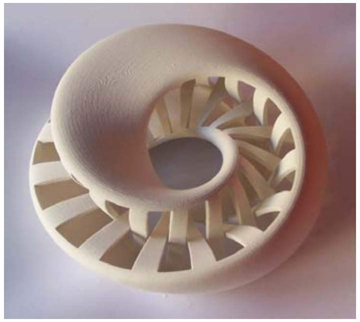
Rapid Prototyping and *3D printing* techniques

Manufacturing techniques that allow the realization of computer-defined *3D* parts through a **layer-by-layer** constructive approach.

RP started in 1987 (*3D Systems, Inc*.)

Parts are used as prototypes for study and analysis, for esthetic and ergonomic check.

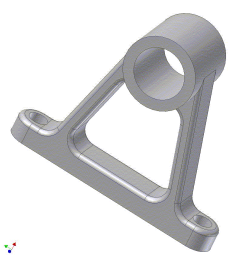
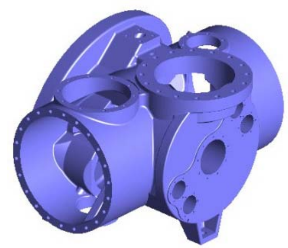
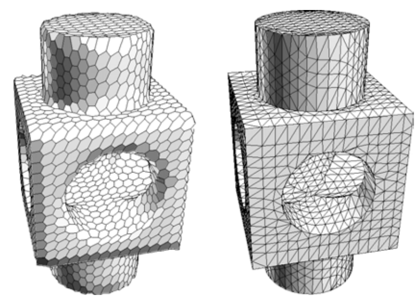
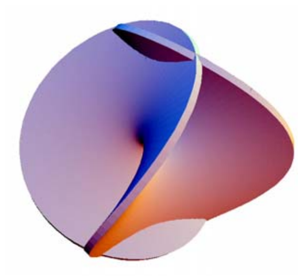
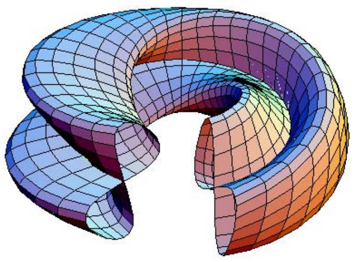
The additive approach widens the range of realizable objects with respect to the traditional milling machines, which *subtract* material from a full block.


Are all *3D* models realizable?

3D printing allows the passage from virtual *3D* models to physical objects.

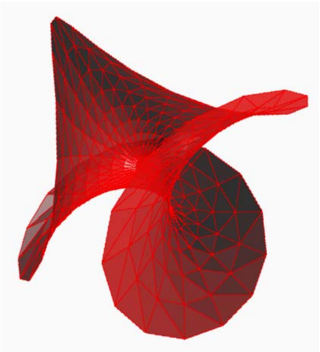
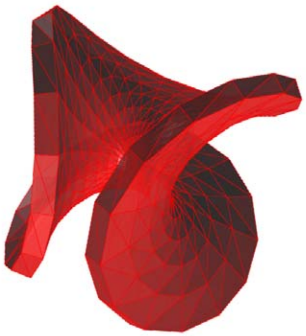
A virtual model can be used as an input file for a *RP* system only if it represents a *real solid.*

Models of surfaces with *RP*

How to use *RP* technique to realize models of parametrized surfaces?

- *RP* deals with solids;
- surfaces in general are not solids;
- we need to:
  - associate a *solid body* to a surface;
  - express it in a suitable way for *RP* systems;
  - correct *B-Rep* incoherence;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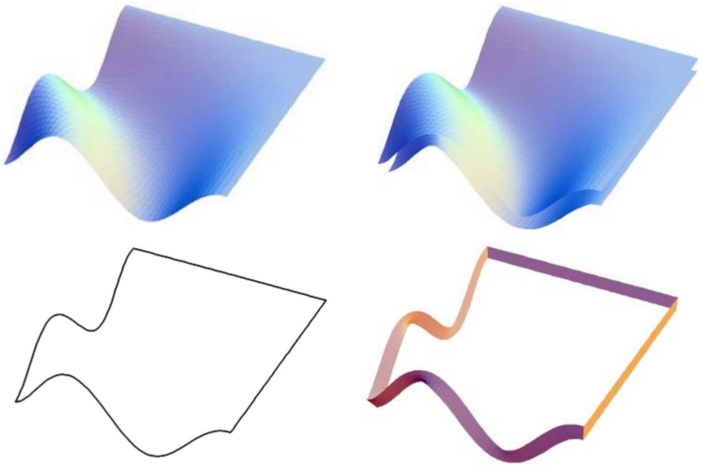
From a surface to a solid body

Giving thickness to an open surface patch

$\mathbf{X}$: K - - > $\mathbf{R}^3$

$$\mathbf{X}(u,v)\begin{cases} x(u,v) = u \\ y(u,v) = v \\ z(u,v) = \frac{1}{2}\cos(uv) \end{cases}$$



$$\mathbf{X}_a(u,v) = \mathbf{X}(u,v) + a\mathbf{N}(u,v)$$

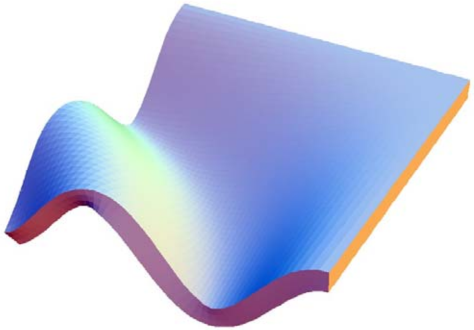
$$\mathbf{X}_{-a}(u,v) = \mathbf{X}(u, v_1 + v_2 - v) - a\mathbf{N}(u, v_1 + v_2 - v)$$

$$\mathrm{bordo}_1(v,t) = \alpha_1(v) - t\mathbf{N}(u_1, v), \quad v \in (v_1, v_2), \quad t \in (-a, a)$$

$$\mathrm{bordo}_2(v,t) = \alpha_2(v) + t\mathbf{N}(u_2, v), \quad v \in (v_1, v_2), \quad t \in (-a, a)$$

$$\mathrm{bordo}_3(u,t) = \alpha_3(u) + t\mathbf{N}(u, v_1), \quad u \in (u_1, u_2), \quad t \in (-a, a)$$

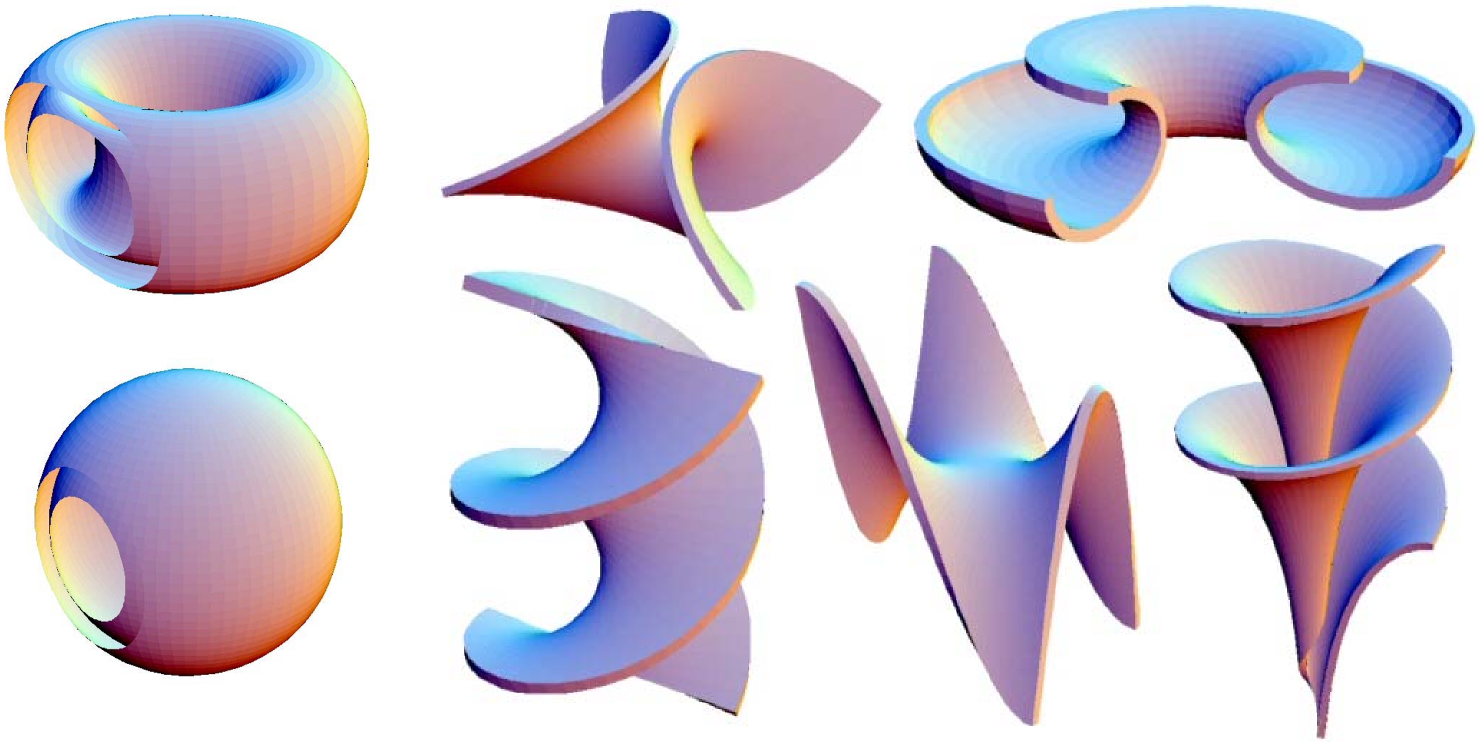
$$\mathrm{bordo}_4(u,t) = \alpha_4(u) - t\mathbf{N}(u, v_2), \quad u \in (u_1, u_2), \quad t \in (-a, a)$$



Final result.

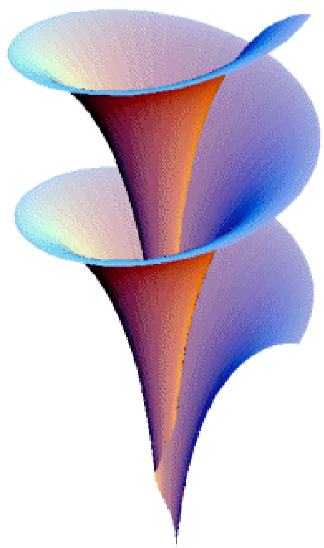
Examples of thickened models

Thickened embedded surfaces

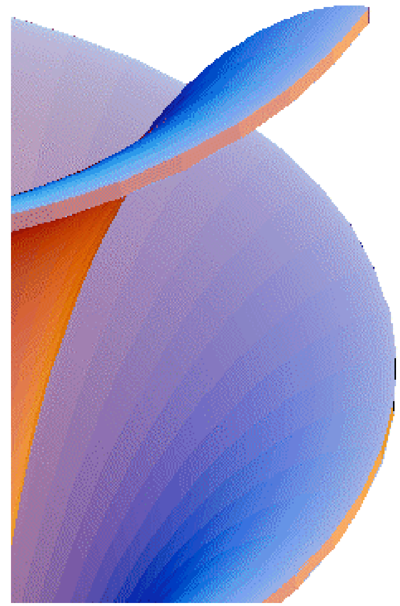


Model of Dini's surface:

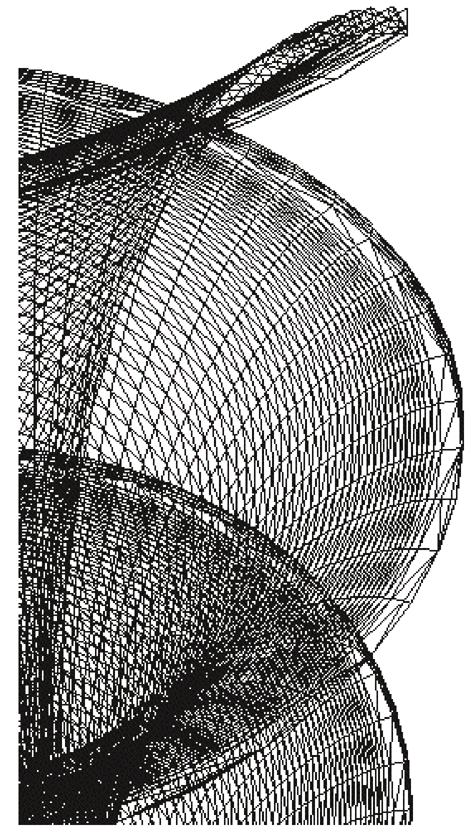
From the mathematical definition to a material model (1/2)



$$\mathbf{dini}(u,v)\begin{cases} \mathrm{a}\cos(u)\sin(v) \\ \mathrm{a}\sin(u)\sin(v) \\ \mathrm{a}(\cos(v)+\log[\tan(\frac{v}{2})])+\mathrm{b}u \end{cases}$$



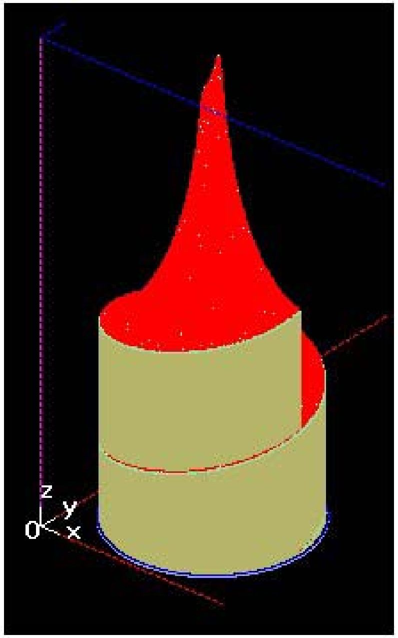
Thickened surface



Mathematical model
in *STL* format

Model of Dini's surface:

From the mathematical definition to a material model (2/2)



Building simulation;

Building process;

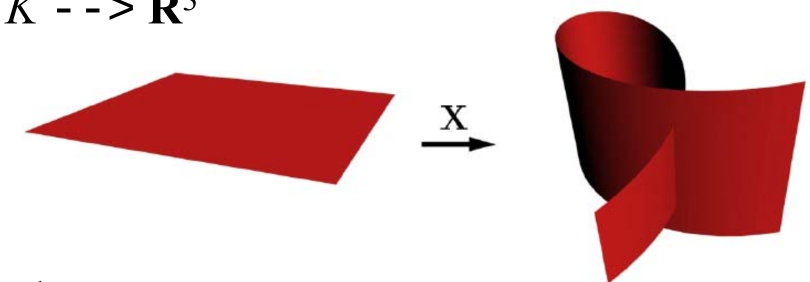
Final result;

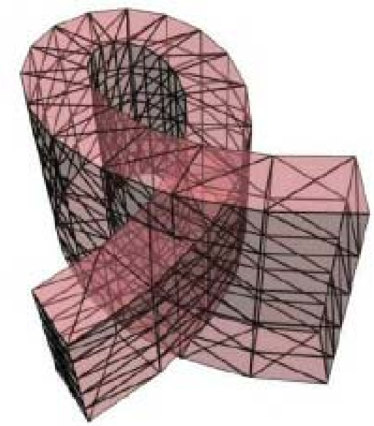
Painted model;

From a surface to a solid body

Giving thickness to self-intersecting surfaces

X: *K* - - > **R**^{3}





$$\mathbf{X}(u,v) = \begin{cases} (6\cos u + 4)\cos u \\ (6\cos u + 4)\sin u \\ \frac{1}{2}uv \end{cases}$$

$$u \in [21\pi/32, 43\pi/32], \;\; v \in [-1/2, 1/2]$$

- same thickening process as in the previous case;

- the resulting object *does not* represent a solid;

- some corrections are needed;

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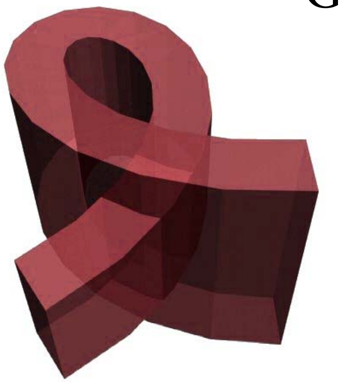
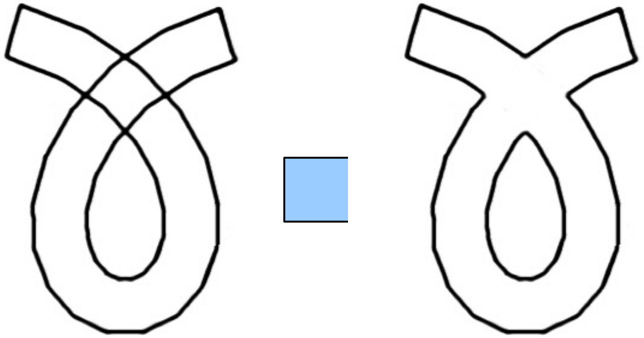
From a surface to a solid body

Giving thickness to self-intersecting surfaces



- we switch to a sliced representation of such non-valid objects;

- find the problems on each single slice and correct it;

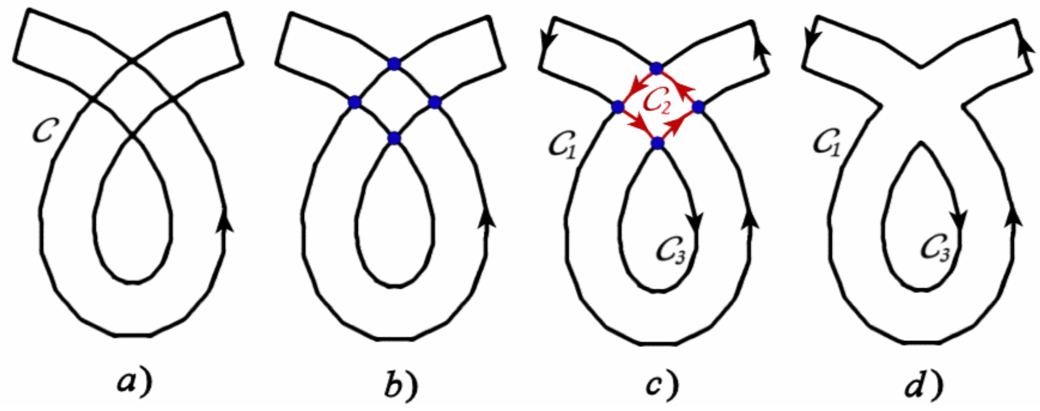


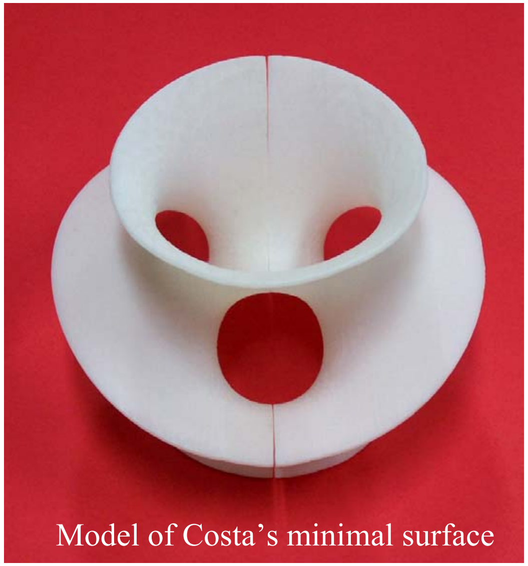
From a surface to a solid body

Giving thickness to self-intersecting surfaces

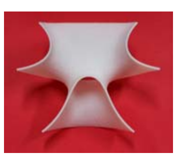
- as a final result we have a collection of planar sections which identifies a valid solid object;

- in order to get this result we used both results from **differential geometry** (normal vector field on regular surfaces, Jordan-Brouwer theorem) and from **computational geometry** (boolean operations on plane domains defined by polygonal lines, overlapping contours)





Model of Costa's minimal surface



Conclusions

We developed a pipeline which has as **starting step** the mathematical definition of a *compact regular immersed surfaces* and as **final step** the ready-to-print solid model.

Future work

- getting rid from some bugs coming out on representations with a high number of polygons;

- going back from the sliced representation to a correct boundary representation of the solid object;

Thank you
for attention.